The Added Value of Convection-permitting Simulations for Dynamically Downscaling Precipitation Extremes

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[2]Department of Physics, University of Toronto

September 4th, 2018

Outline

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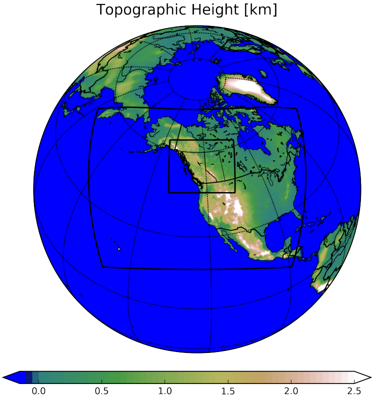
Dynamical Downscaling using WRF

The Weather Research and Forecast model (WRF) is a state of the art NWP model and can be used as a regional climate model.



Topography and WRF Domains

WRF : 30 km, 10 km & 3 km over North America

ERA-I : $\sim 0.7^\circ \approx 70$ km resolution

CESM : $\sim 1^\circ \approx 100$ km resolution

Boundary Conditions

- ERA Interim Reanalysis
- CESM Climate Model (for projection)

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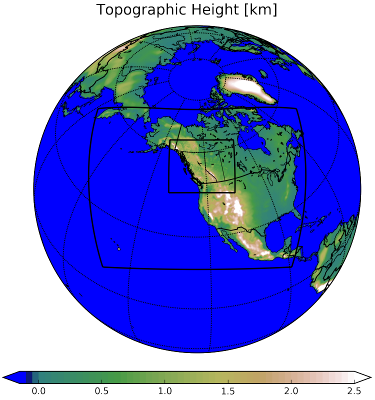

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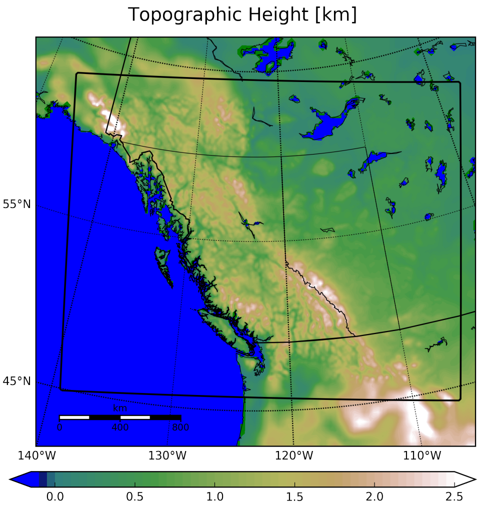
High Resolution Climate Simulations

WRF Domains

- 10 km Western Canada
- 3 km BC & Alberta

- Non-hydrostatic, finite-difference dynamical core
- Noah LSM, RRTMG radiation, Morrison-2-Moment MP, Grell-3 cumulus (except 3 km)



Intermediate WRF domain (10 km)
and inner-most WRF domain (3 km)

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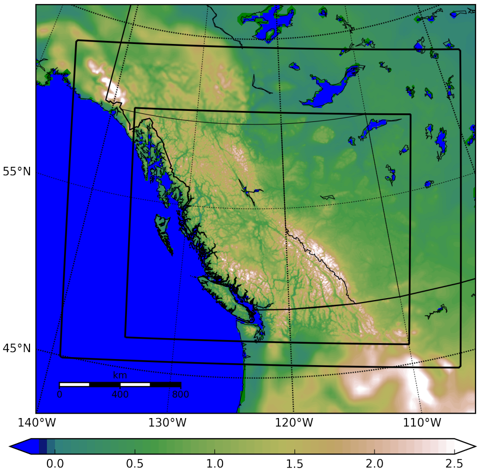
Topographic Height [km]



Intermediate WRF domain (10 km) and inner-most WRF domain (3 km)

Outline

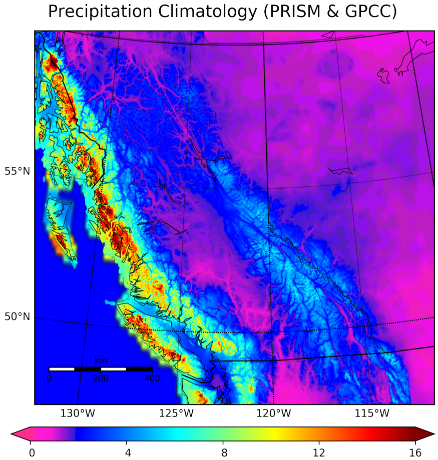
Introduction: Dynamical Downscaling
- Models and Experimental Setup

Very High Resolution Climate Simulations
- Observed and Modeled Precipitation

Extreme Value Analysis
- Clustering of Station Observations
- Dependence on Model Resolution

End-of-Century Precipitation Projections
- The Role of Convective Precipitation

Western Canada: Orographic Precipitation



Annual average precipitation over Western Canada (Merged Dataset)

General precipitation pattern

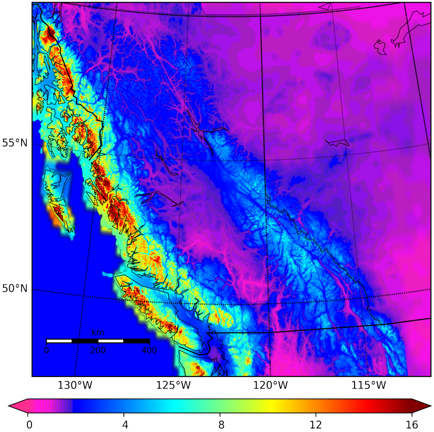
Merged Observations

Combined data from PRISM (1 km) and GPCC (0.25°)

- Very high precipitation in the Coast Mountains
- Interior Plateau and the lee of the Rockies are very dry
- Precipitation gradient stronger in winter, weaker in summer

Western Canada: Orographic Precipitation

Precipitation Climatology (PRISM & GPCC)



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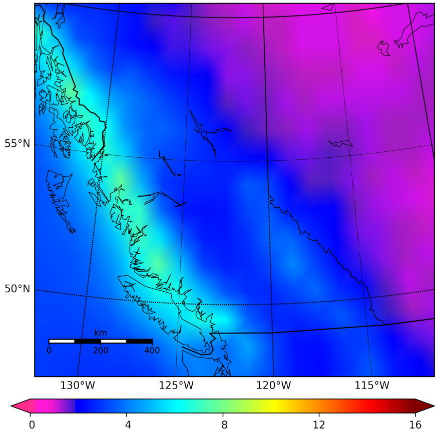
CESM (100km, 60yr)



Annual average precipitation over Western Canada (CESM at 100 km)

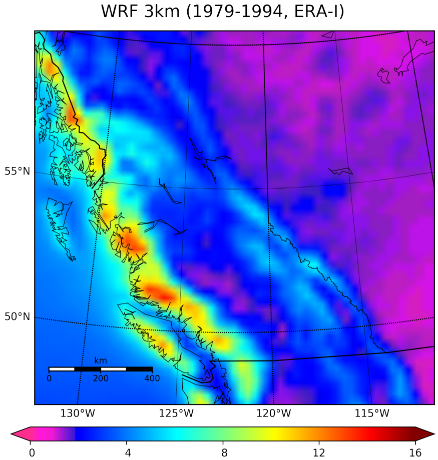
Dependence on Resolution

Interior Plateau

At GCM resolution the Interior Plateau is almost entirely smoothed out

- At 30 km the Interior Plateau and Coast Mountains appear
- At 10 km the Rocky Mountain Trench emerges
- At 3 km elevations over 3000 m are resolved, and rain shadows improve

Western Canada: Orographic Precipitation



Annual average precipitation over Western Canada (WRF at 30 km)

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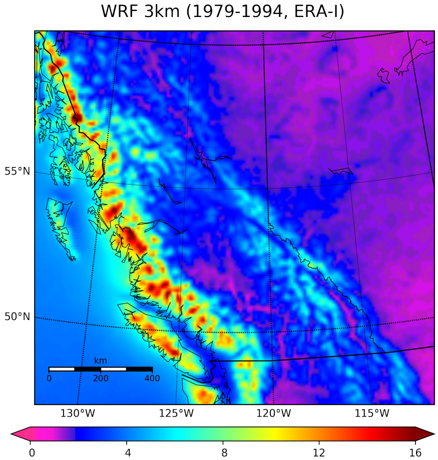

Annual average precipitation over Western Canada (WRF at 10 km)

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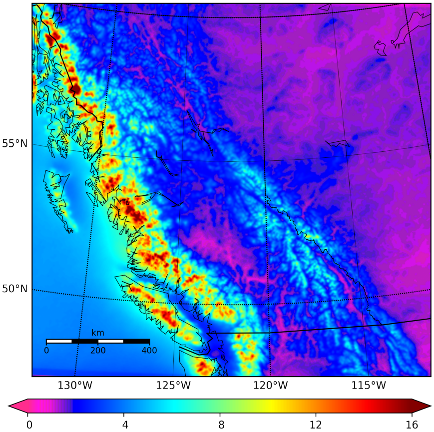
Western Canada: Orographic Precipitation

WRF 3km (1979-1994, ERA-I)



Annual average precipitation over Western Canada (WRF at 3 km)

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Mean Precipitation Bias by Resolution

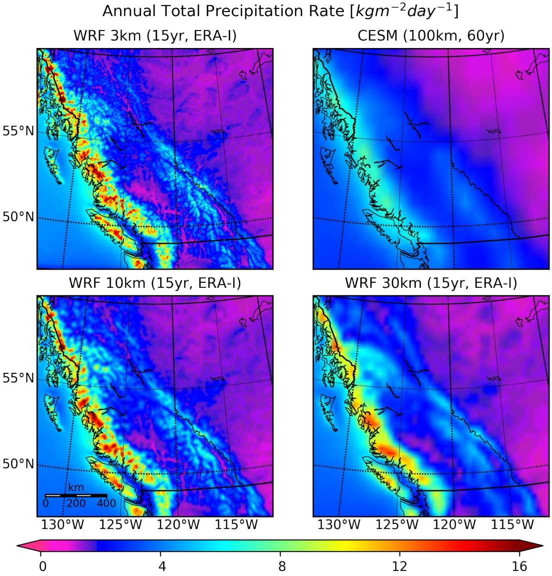
Annual Precipitation

- Rain shadow improves with resolution...
- ... but is still underestimated

Large Biases

A systematic positive biases exists on top of inadequate resolution

CESM generally has a negative bias in summer.



Mean Precipitation Bias by Resolution

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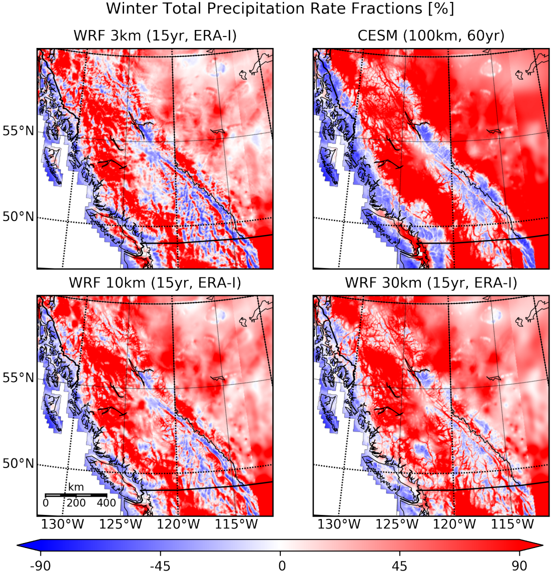

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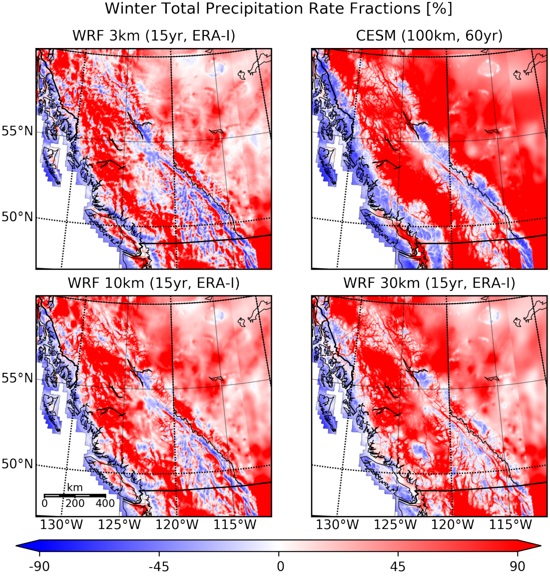

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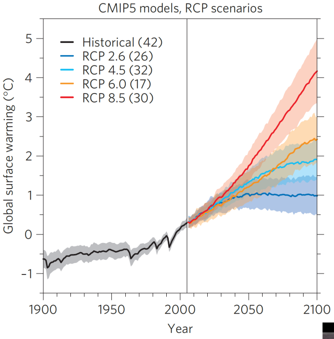
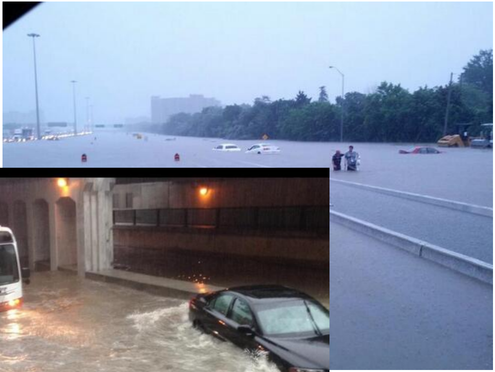


IPCC AR5 (2013) projections for global surface temperature under different scenarios.

Precipitation Extremes

A major concern with regard to Climate Change is the potential increase in extreme weather events.

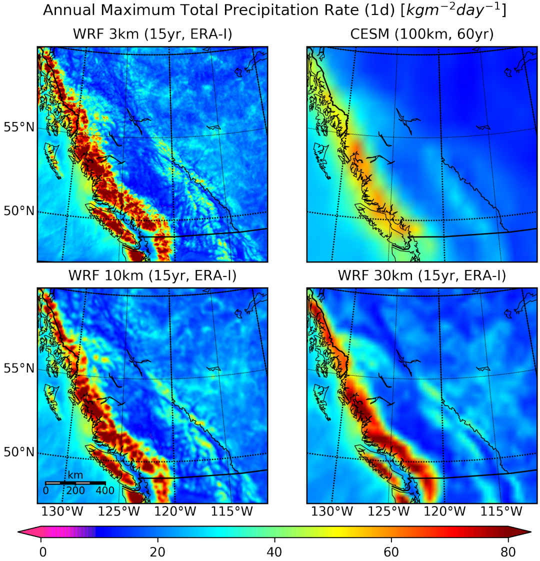


Toronto Flood, Summer 2013

Rare Events

Statistically detecting trends in extremes is difficult due to rarity of events (by definition).

Extreme Precipitation Climatology (Models Only)



Annual Maximum of Daily Precipitation

- Spatial pattern generally follows mean precipitation
- Variability is much higher than in mean

No Observations

There are no gridded observational estimates for precipitation extremes

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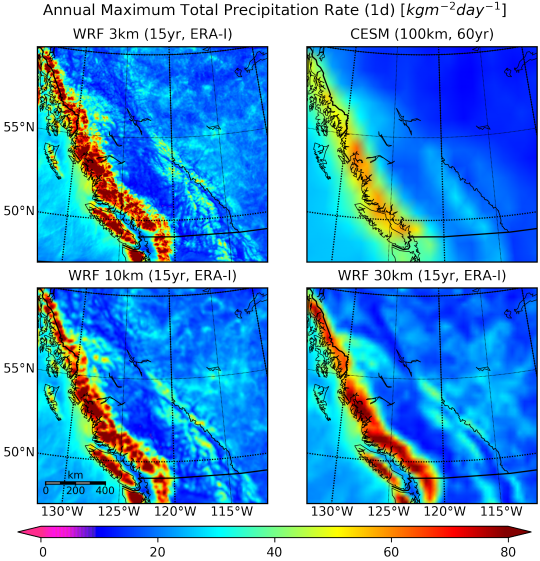

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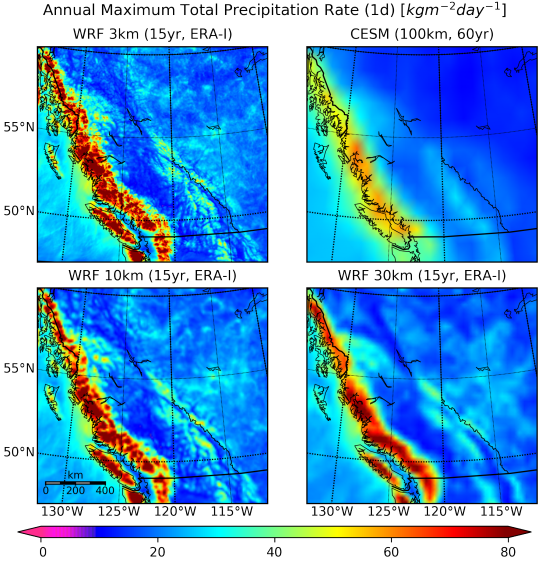

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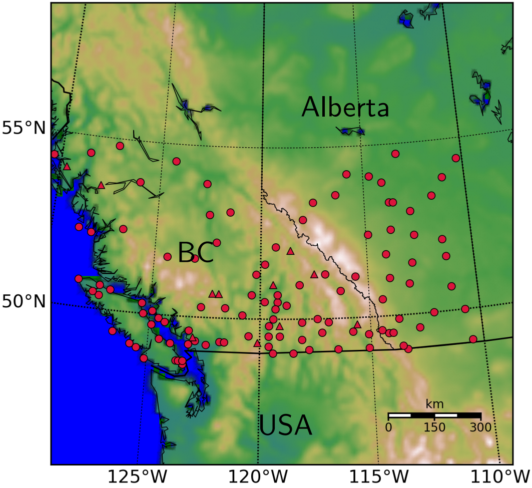
Rain Gauges in Western Canada

Used for Analysis of Precipitation Extremes

Rain Gauges



Environment Canada Weather Station, Mekis & Vincent (2011, *AO*)

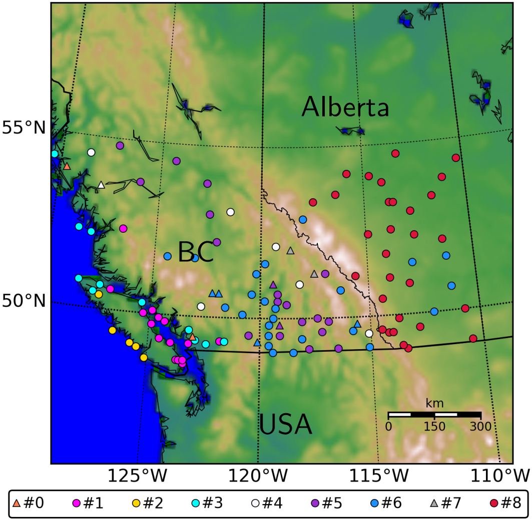
Station Clustering

For Analysis of Precipitation Extremes

Station selection

- 12 stations in the Pacific Coast cluster
- 30 stations in the Prairies (AB) cluster

Stations were clustered based on similarity of precipitation climatology



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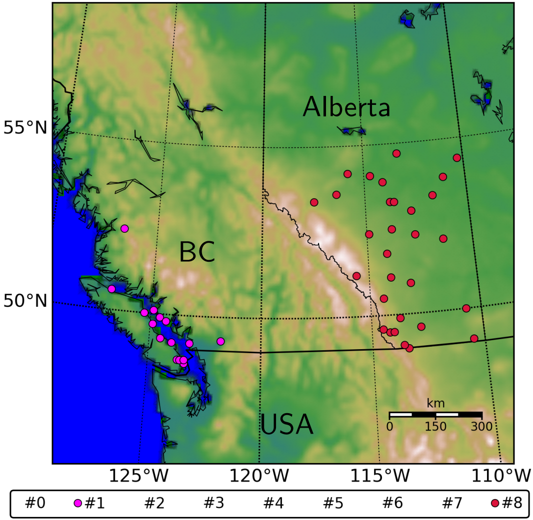

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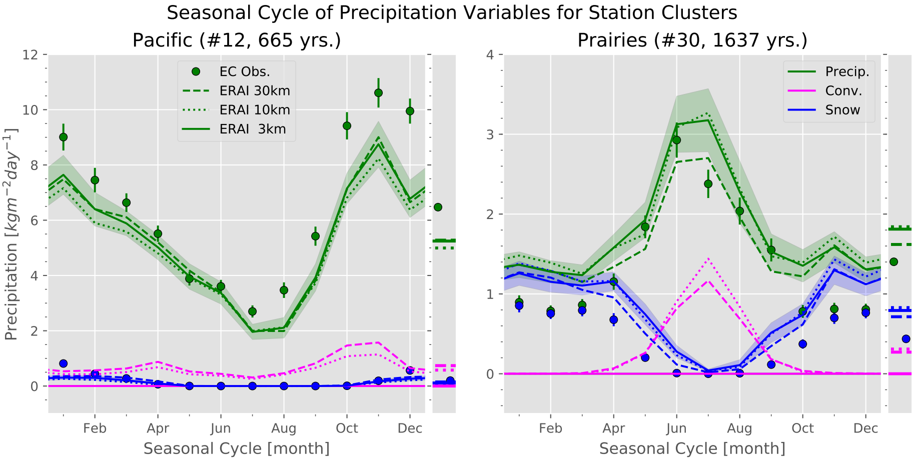

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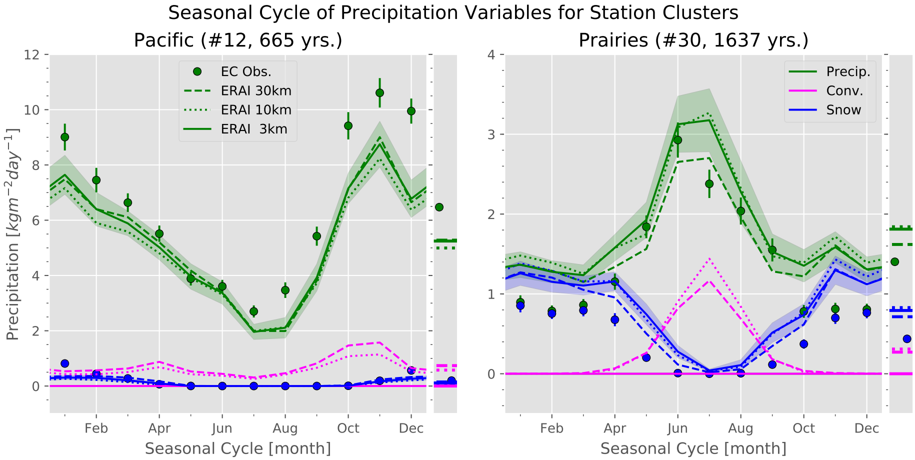
- 3 km WRF has best seasonal cycle, especially in Prairies



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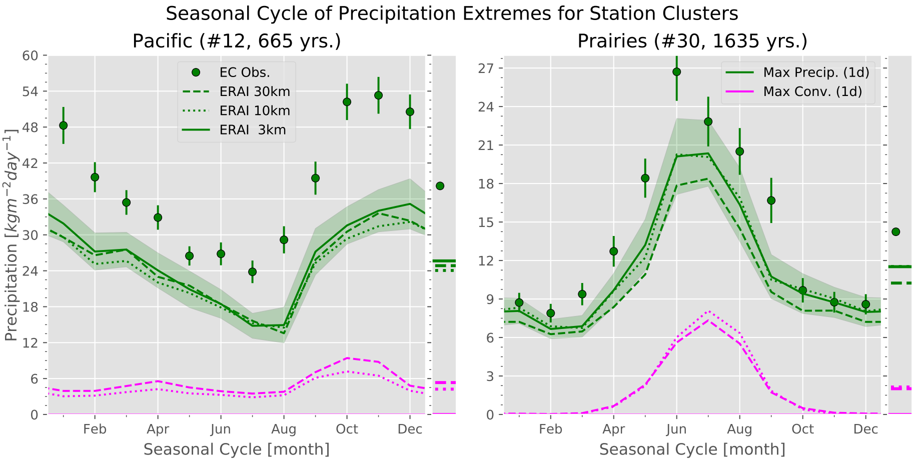

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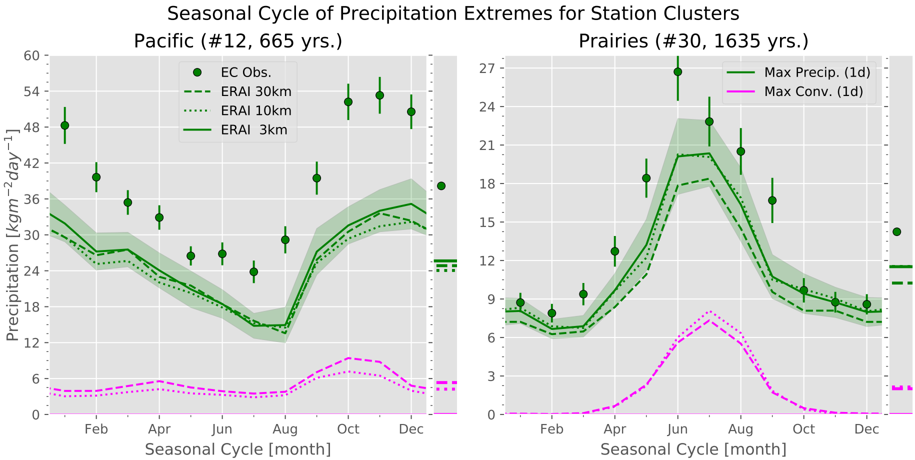

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Extreme Value Analysis: the GEV Distribution

Extreme Value Theorem

The GEV is the limit-distribution of the block-maxima of any randomly distributed variable

- Distribution of seasonal precipitation maxima

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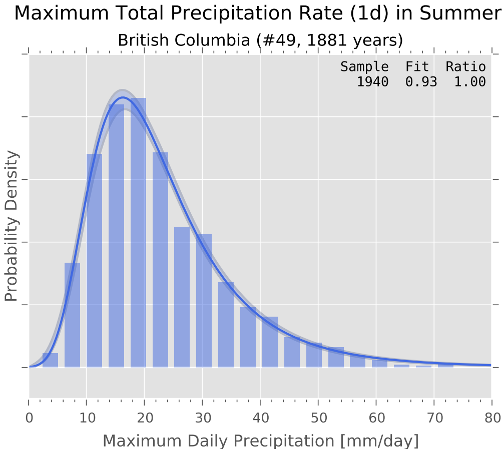
Samples are pooled over all stations within a cluster and over all years



Pooled summer precipitation maxima over all of BC. 95% CI (bands) from bootstrapping.

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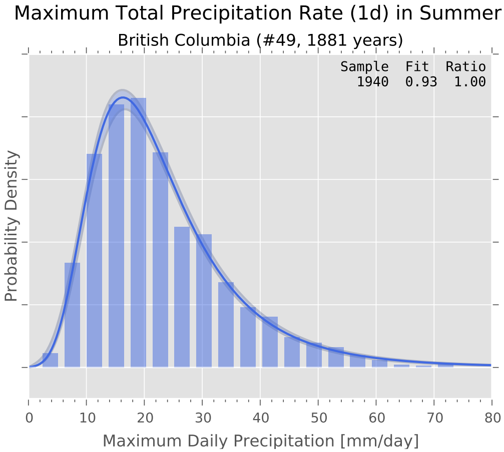

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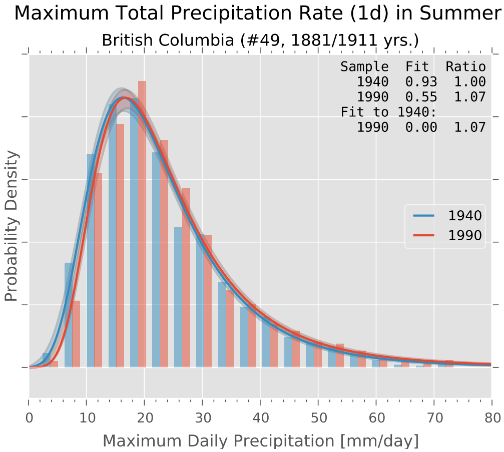
Extreme Value Analysis: the K-S Test

The K-S Test is used ...

- ... as a measure for Goodness of Fit
- ... to test the significance of differences (to observations)

The K-S Test

The *Kolmogorov-Smirnov* (K-S) Test measures the statistical significance of differences between two distributions



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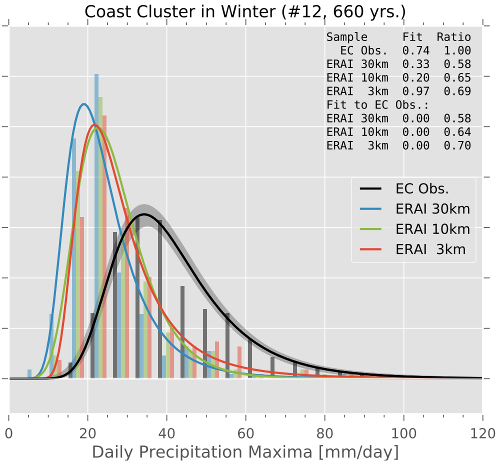
Precipitation Extremes: Linear Rescaling



EC Stations & WRF Simulations
at the Pacific Coast in Winter

Winter Extremes for the Pacific Coast Cluster

- GEV fits histograms reasonably well
- WRF intensity is consistently 60 - 70% of Observations

Linear Rescaling

Simple linear rescaling of the mean results in good fit to Observations!

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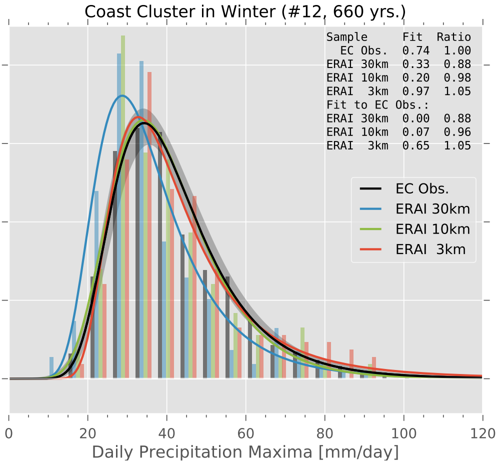

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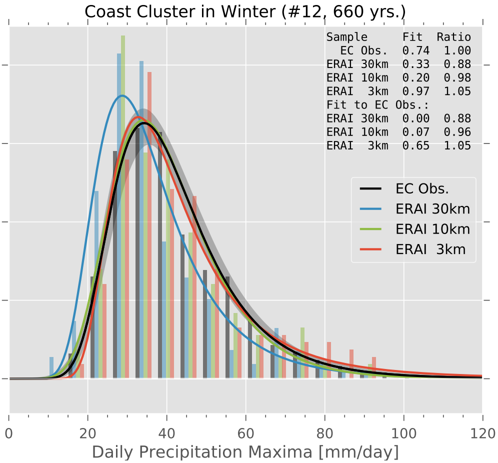

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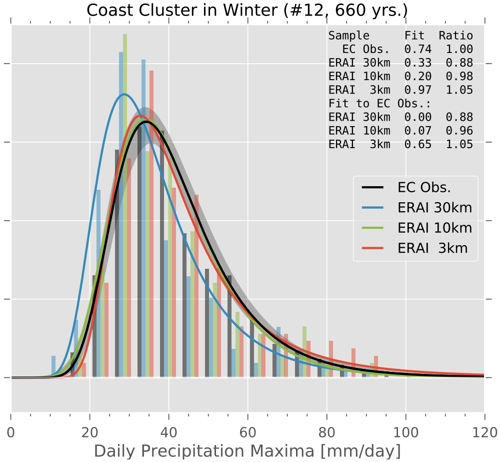

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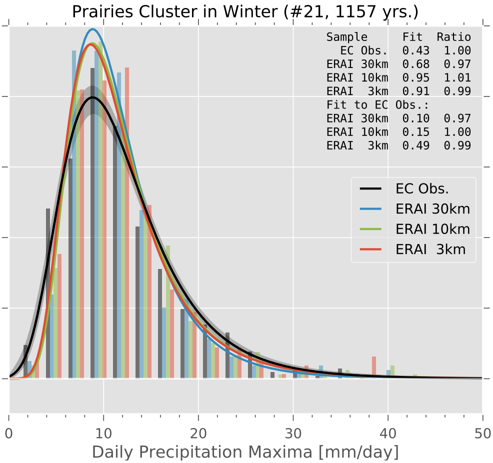

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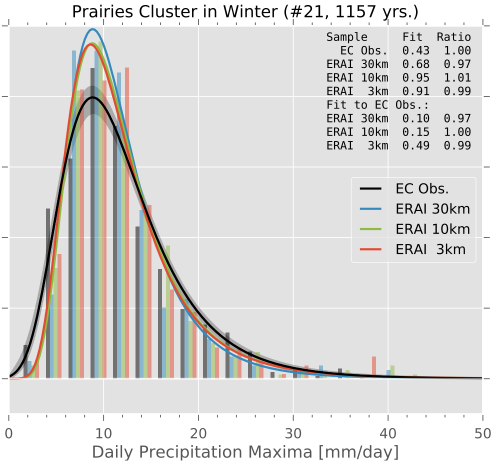

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Summer Extremes for the Prairies Cluster

No Added Value?

Difference between parameterized and resolved convection is small at best!

- In summer, resolution does not seem to matter that much
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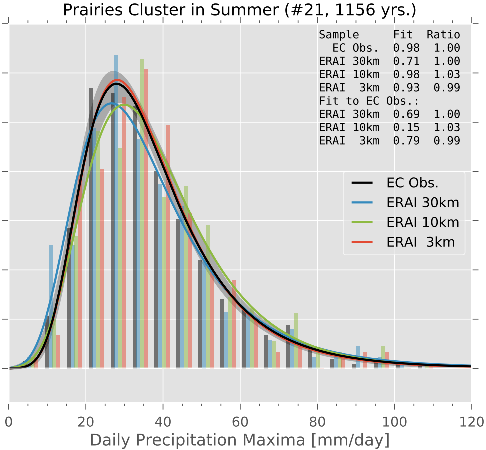

EC Stations & WRF Simulations in the Prairies in Summer

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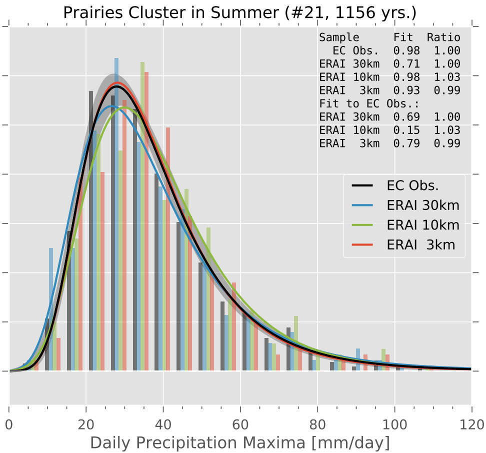

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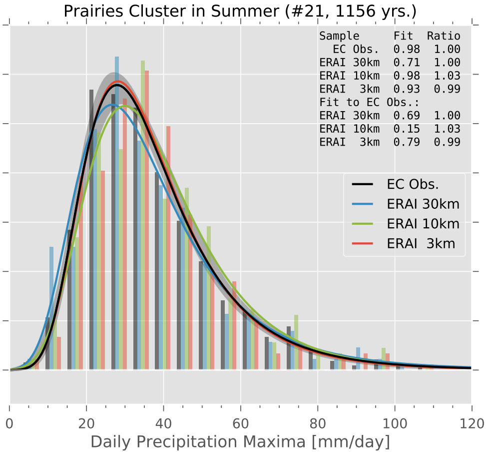


EC Stations & WRF Simulations in the Prairies in Summer

Outline

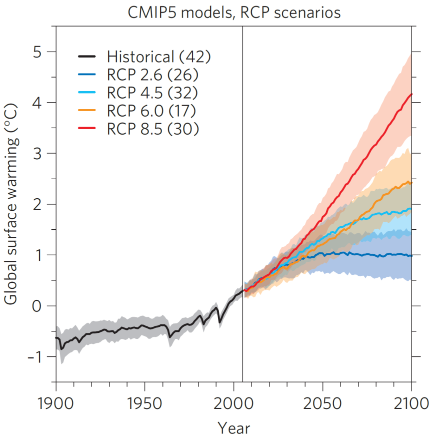
Regional Climate Projections

WRF Ensembles

- 3 km end-century projections in progress (15 / 7 years)
- **Two WRF initial condition ensembles (60 years)** with different cumulus schemes

Experimental Setup (30/10 km)

- GCM & RCM projections
  - Historical (1979 - 1994)
  - Mid-21st-Century (2045-2060)
  - End-21st-Century (2085-2100)
- only RCP 8.5 GHG scenarios



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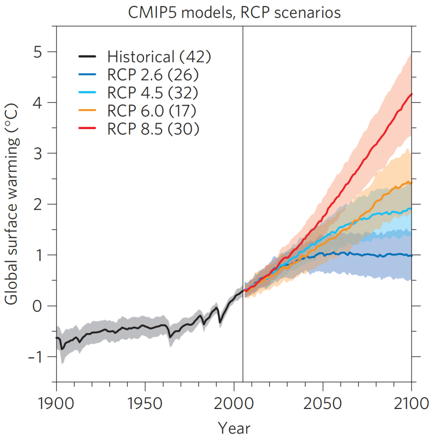

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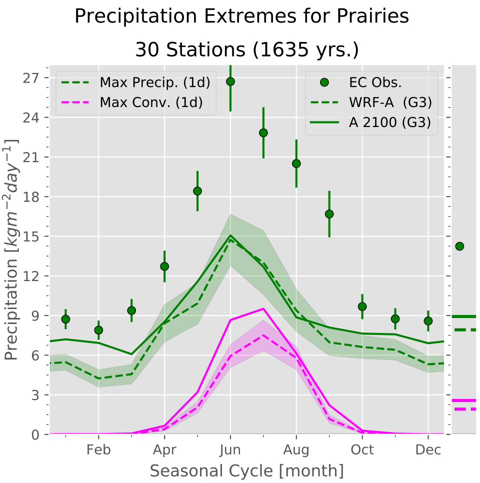
Uncertainty in Convective Precipitation

Total and Convective Precip Extremes (over the Prairies)

- Convective precip extremes increase, ...
- ... but total precip increase depends on convective fraction

Cumulus Scheme

Change in summer precip extremes is sensitive to cumulus scheme



WRF at 30 km with Grell-3 cumulus scheme

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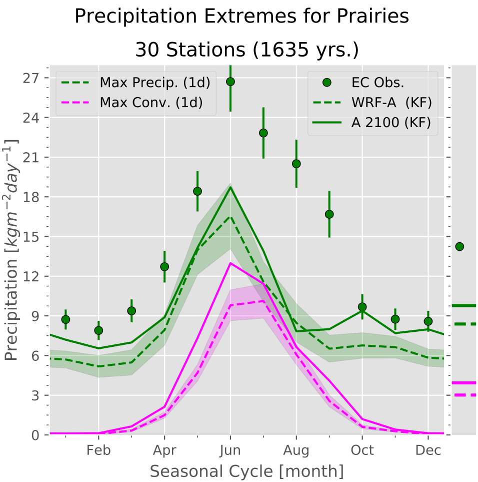

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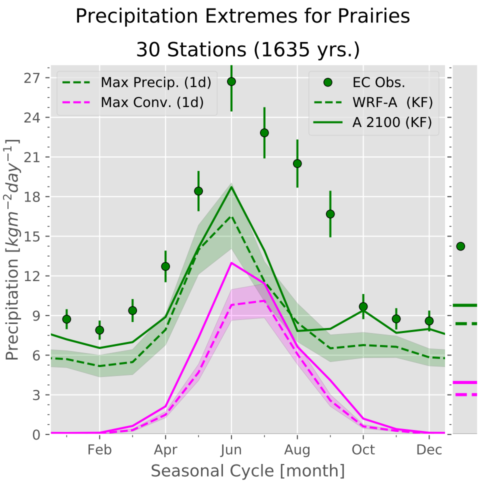

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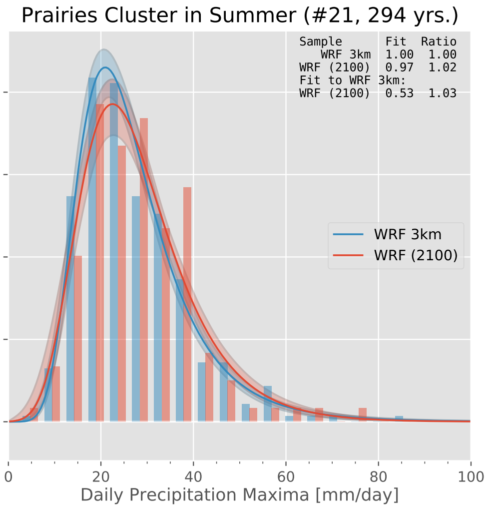
Summer Extremes: Convection-Permitting Projections

Summer Extremes for Prairies (Projections)

- No statistically significant differences, ...
- ... but high uncertainty due to short period

Preliminary Results

Convection permitting simulations show no increase in summer extremes



3 km WRF simulations for historical and end-century periods (15 & 7 years)

Summer Extremes: Convection-Permitting Projections

Summer Extremes for Prairies (Projections)

- No statistically significant differences, ...
- ... but high uncertainty due to short period

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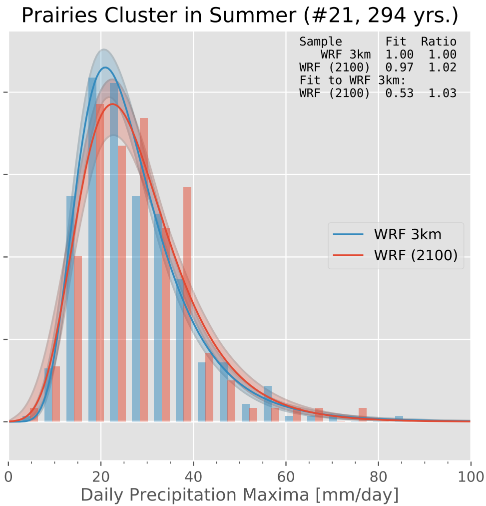

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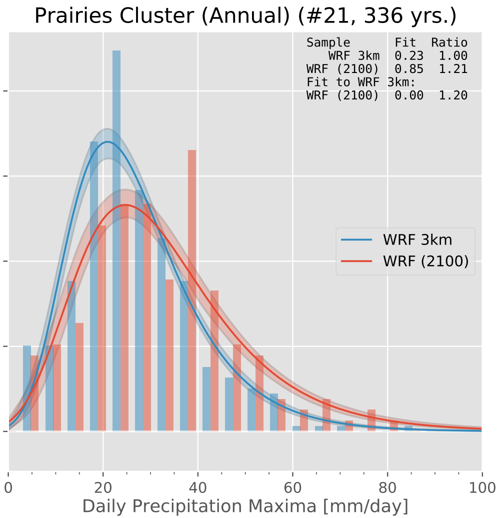
Projection for Annual Precipitation Extremes



3 km WRF simulations for historical and end-century periods (15 & 7 years)

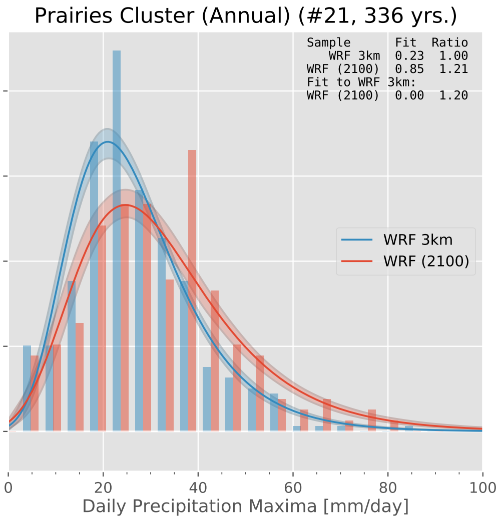
Annual Extremes for Prairies (Projections)

- Clear and highly significant increase
- Sample size is sufficient for a strong signal
- The increase primarily driven by and increase in spring

Poor GEV Fit

GEV for annual extremes does not fit as well, and differs from observations

Projection for Annual Precipitation Extremes



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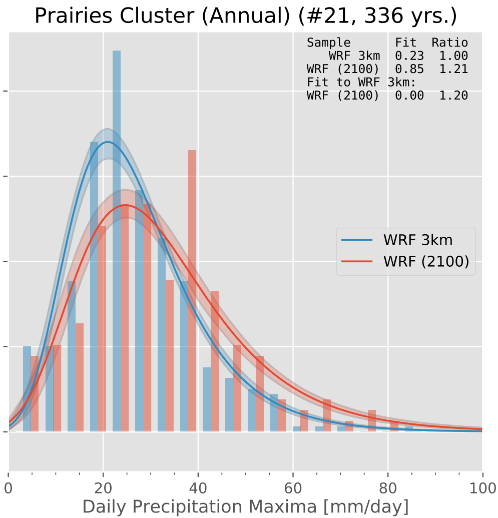

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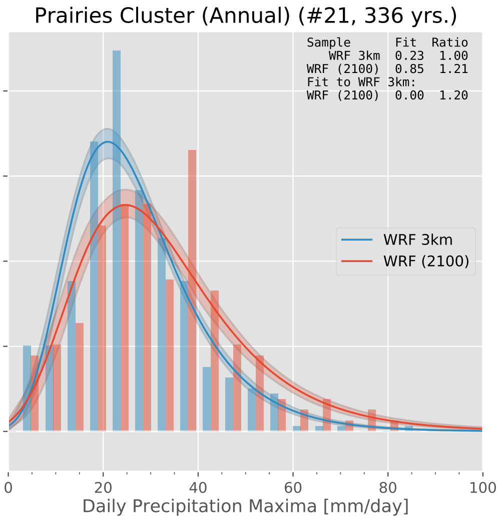

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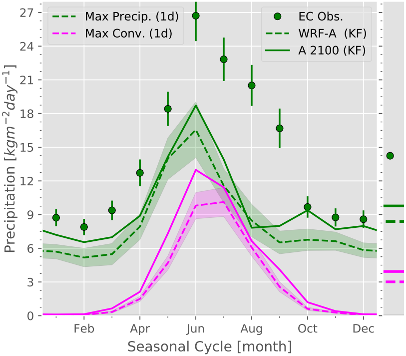
Summary of Results

- WRF has a large precipitation bias, regardless of resolution
- But extremes are consistently underestimated

The Added Value of High Resolution

- In winter higher resolution affords a much better distribution fit
- In summer even parameterized convection fits the observations...
- ... but resolving convection reduces model uncertainty for summer precip projections



Uncertainty in convective precip

- Robust increase in winter precipitation (extremes), but shift in warm season

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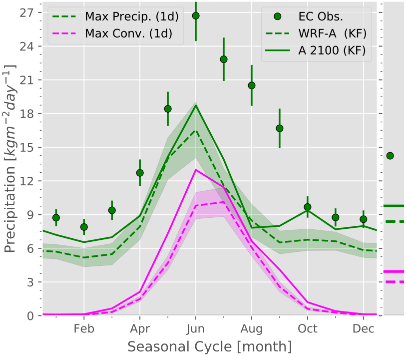
Precipitation Extremes for Prairies
30 Stations (1635 yrs.)



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Thank You! ∼ Questions?

Acknowledgements:

Dr. Marc d'Orgeville

Dr. Jonathan Gula

List of Relevant Publications

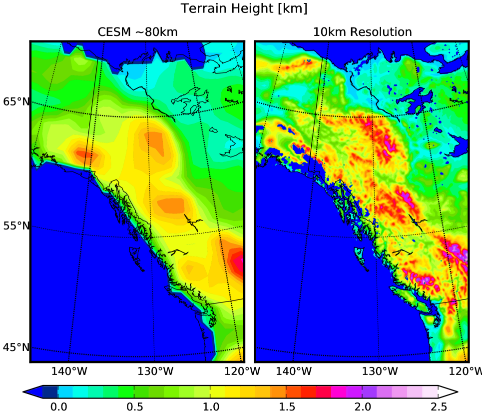
- **Erler, Andre R.** and W. Richard Peltier, 2016, "Projected Changes in Precipitation Extremes for Western Canada based on High-resolution Regional Climate Simulations", *Journal of Climate*.
- **Erler, Andre R.** and W. Richard Peltier, 2017, "Projected Hydro-climatic Changes in Two Major River Basins at the Canadian West Coast Based on High-resolution Regional Climate Simulations", *Journal of Climate*.
- **Erler, Andre R.**, W. Richard Peltier and Marc d'Orgeville, 2015, "Dynamically Downscaled High Resolution Hydro-Climate Projections for Western Canada", *Journal of Climate*.
- Peltier, W. Richard, Marc d'Orgeville, **Andre R. Erler** and Fengyi Xie, 2018, "Uncertainty in Future Summer Precipitation in the Laurentian Great Lakes Basin: Dynamical Downscaling and the Influence of Continental Scale Processes on Regional Climate Change", *Journal of Climate*.
- Marc d'Orgeville, W. Richard Peltier and **Andre R. Erler**, Jonathan Gula, 2014, "Climate change impacts on Great Lakes Basin precipitation extremes", *Journal of Geophysical Research*.

WRF: Climate Projection at 10 km Resolution

High Resolution

Complex coast line & mountains of western Canada are poorly resolved in CESM

- Non-hydrostatic, finite-difference dynamical core
- Noah LSM, RRTMG radiation, Morrison-2-Moment MP, Grell-3 cumulus
- Also coupled a simple lake model (FLake)



Topography of Western Canada.
Left: CESM at ∼80 km
Right: WRF (inner domain) at 10 km

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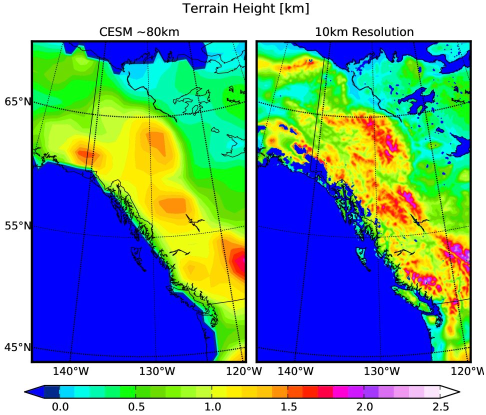

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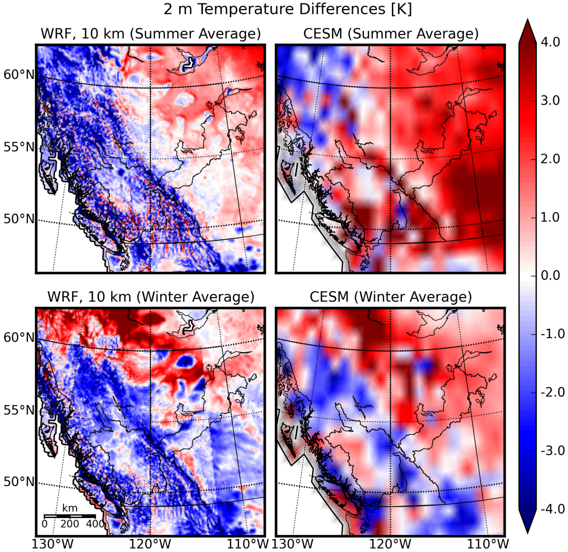
Validation: Temperature

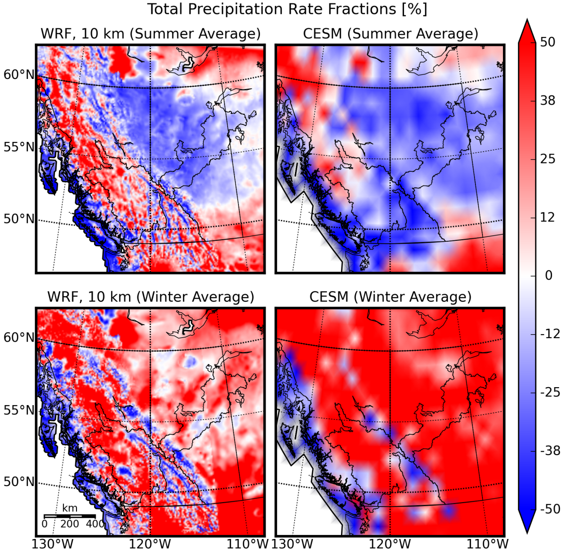


Ensemble Average
(differences w.r.t. merged observations)

- CESM has warm bias in summer and weaker seasonal cycle
- Mountains appear to be too cold and plains too warm

WRF also has a significant cold bias in spring due to poor modeling of snow

Validation: Precipitation



Ensemble Average

(fractions w.r.t. merged observations)

- First rain barrier significantly underestimated (resolution dependent)
- Precipitation in rain shadow overestimated
- Seasonal cycle in lee of Rockies under-estimated (less so in WRF)

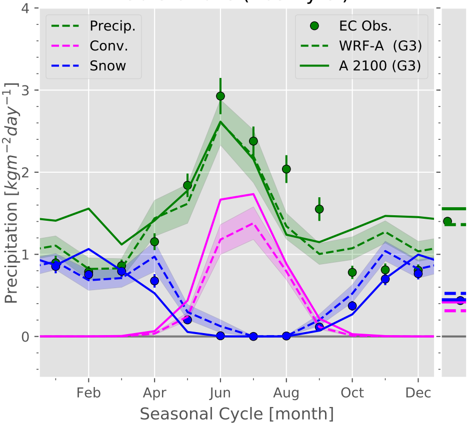
Uncertainty in Convective Precipitation

Total and Convective Precip over the Prairies

- Convective precip increases robustly, ...
- ... but total precip increase depends on convective fraction

Cumulus Scheme

Summer precipitation increase is sensitive to cumulus scheme



WRF at 30 km with Grell-3 cumulus scheme

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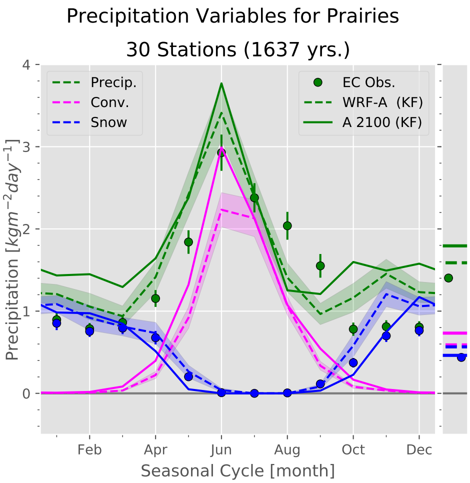

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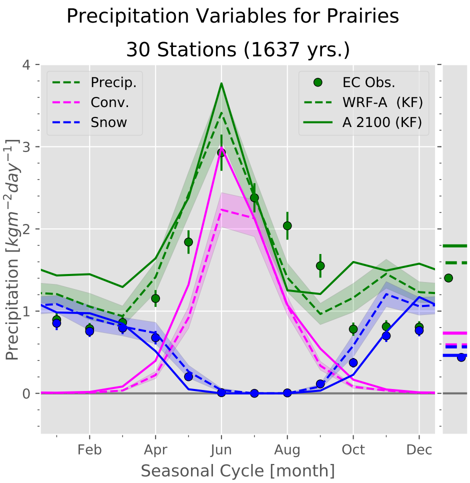

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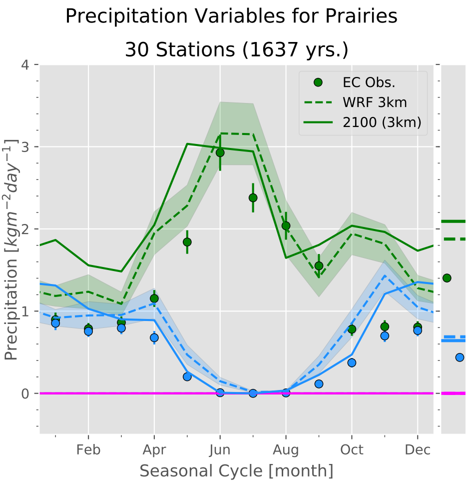
Precipitation Changes at High Resolution (3 km, 2100)



WRF at 3 km (Monthly Normals)

Total and Extreme Precipitation (Prairies)

- Increase in winter and spring, ...
- ... but not in summer
- Same response in precip extremes

Natural Variability

Note that this is only based on 15 & 7 years of simulation, and decadal variability is significant in western Canada!

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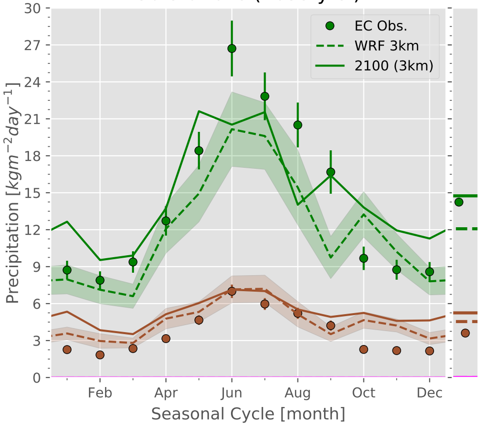

WRF at 3 km (Monthly Extremes)

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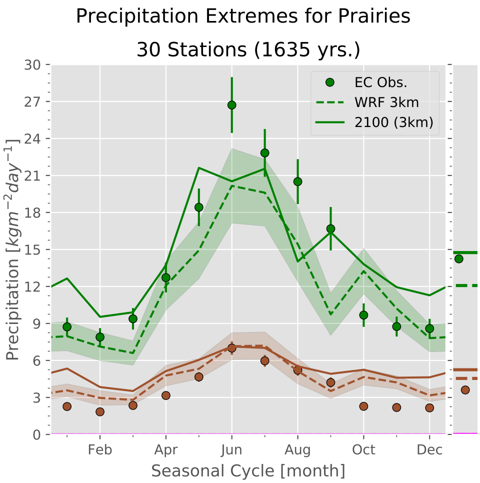

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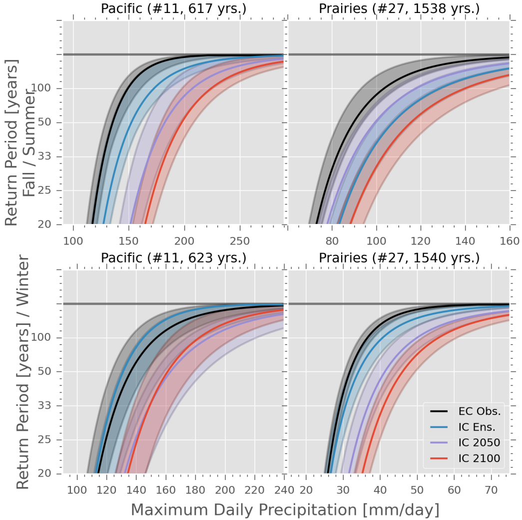
Extreme Quantiles and Return Periods



Tail CDF: Observations & Projections (rescaled)

Intensity of Decadal Return Period Events

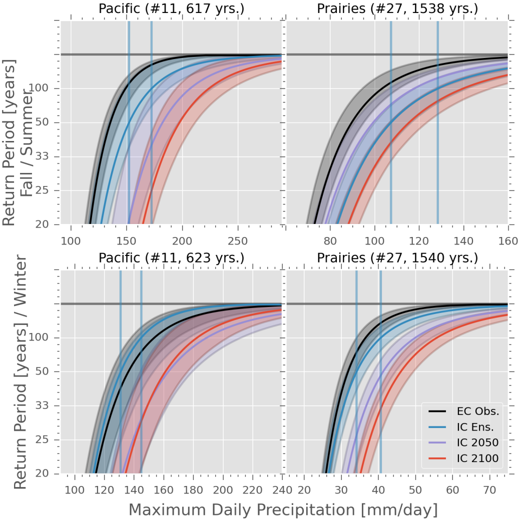
- Extreme tails overestimated in WRF
- Return periods:

Fall:	100 yr ⇒ 25 yr
Winter:	100 yr ⇒ 33 yr
Summer:	100 yr ⇒ 80 yr

Fattening Tails?

Cold season increase due to shift in mean

Extreme Quantiles and Return Periods



Lines: 50 and 100 year return period events

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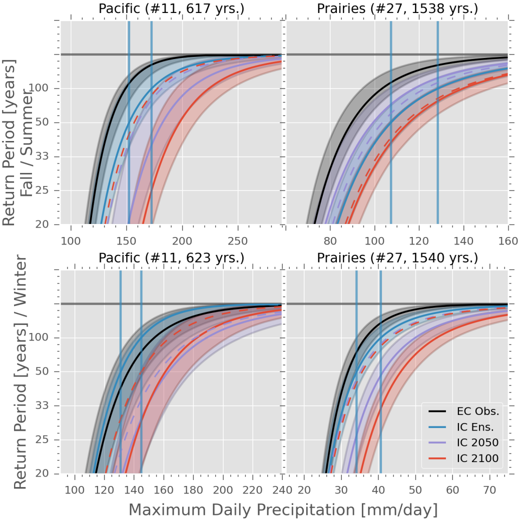

Rescaling of means: relevance of shape change

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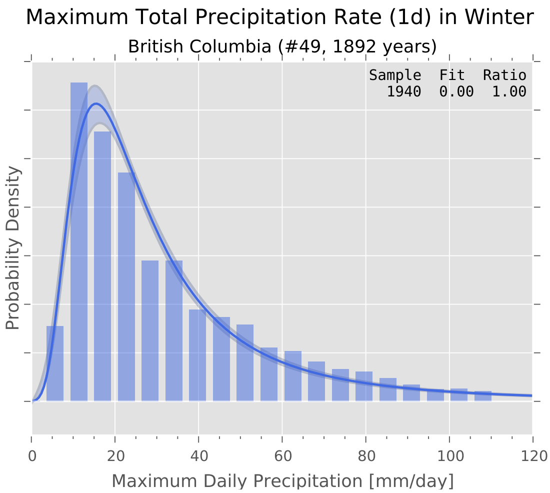
Winter Precipitation Extremes in British Columbia



Pooled winter precipitation maxima over entire province. 95% CI (bands) from bootstrapping.

Distribution of seasonal (winter) block maxima.

Very Bad Fit

Pooled winter extremes do not follow a GEV distribution! (K-S test)

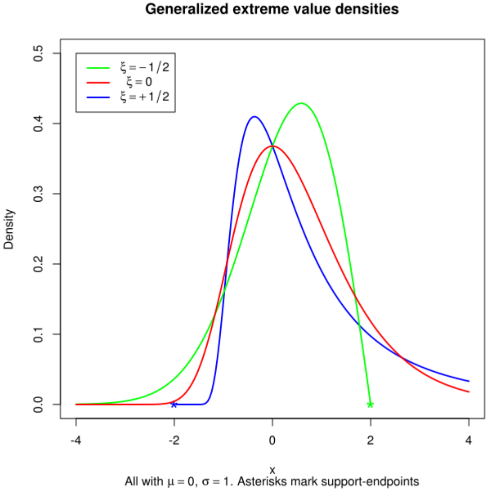
All data were aggregated (pooled) over all 49 stations in BC and a single GEV distribution was fitted.

Extreme Value Analysis: the GEV Distribution

Extreme Value Theorem

The GEV is the limit-distribution of the block-maxima of any randomly distributed variable

- Block length: one season (3 month)
- 3 Parameters: location, scale, shape
- Shape (> 0) controls thickness of tails



"GevDensity" by Created by R D Gill, 4 January 2013, using R script. Licensed under CC BY-SA 3.0 via Wikipedia